A
DISCOURSE
Concerning GRAVITY, *and its Properties, wherein the Deſcent of* Heavy Bodies, *and the Motion of* Projects *is briefly, but fully handled: Together with the* Solution *of a* Problem *of great Uſe in* GUNNERY. *By E. HALLEY.*

NATURE amidſt the great variety of *Problems* wherewith She exerciſes the Wits of Philoſophical men, ſcarce affords any one wherein the Effect is more viſible, and the Cauſe more concealed than in thoſe of the *Phænomena* of *Gravity*. Before we can go alone, we muſt learn to defend our ſelves from the violence of its Impulſe, by not truſting the *Center* of *Gravity* of our Bodies beyond our reach; and yet the Acuteſt Philoſophers, and the ſubtileſt Enquirers into the Original of this Motion, have been ſo far from ſatisfying their Readers, that they themſelves ſeem little to have underſtood the Conſequences of their own *Hypotheſes*.

Des Cartes his Notion, I muſt needs confeſs to be to me Incomprehenſible, while he will have the Particles of his *Celeſtial matter*, by being reflected on the Surface of the *Earth*, and ſo aſcending therefrom, to drive down into their places thoſe *Terreſtrial Bodies* they find above them: This is as near as I can gather the ſcope of the 20, 21, 22, and 23 *Sections* of the laſt Book of his *Principia Philoſophiæ*; yet neither he, nor any of his Followers can ſhew how a Body ſuſpended *in libero æthere*, ſhall be carried downwards by a continual Impulſe tending upwards, and acting upon all its parts equally: And beſides the obſcurity wherewith he expreſſes himſelf particularly, *Sect.* 23. does ſufficiently argue according to his own Rules, the confuſed *Idea* he had of the thing he wrote.

Others, and among them Dr. *Voſſius* aſſerts the Cauſe of the *Deſcent* of *heavy Bodies* to be the *Diurnal rotation* of the *Earth* upon its *Axis*, without conſidering, that according to the Doctrine of Motion fortified with Demonſtration, all Bodies moved *in Circulo*, would recede from the Center of their Motion; whereby the contrary to *Gravity* would follow, and all looſe Bodies would be caſt into the Air in a *Tangent* to the *Parallel* of *Latitude*, without the intervention of ſome other Principle to keep them faſt, ſuch as is that of *Gravity*. Beſides the effect of this Principle is throughout the whole Surface of the Glob found nearly equal, and *certain Experiment* ſeems to argue it rather leſs near the *Equinoctial*, than towards the *Poles*, which could not be by any means, if the *Diurnal rotation* of the *Earth* upon its *Axis* were the cauſe of *Gravity*, for where the Motion was ſwifteſt, the Effect would be moſt conſiderable.

Others aſſign the *Preſſure* of the *Atmoſphere*, to be the Cauſe of this Tendency towards the *Center* of the *Earth*; but unhappily they have miſtaken the *Cauſe* for the *Effect*, it being from undoubted Principles plain, that the *Atmoſphere* has no other *Preſſure* but what it derives from its *Gravity*, and that the Weight of the upper parts of the *Air*, preſſing on the lower parts thereof, do ſo far bend the Springs of that *Elaſtick* Body, as to give it a force equal to the Weight that Compreſſed it, having of it ſelf no force at all: And ſuppoſing it had, it will be very hard to explain the *Modus*, how that *Preſſure* ſhould occaſion the *Deſcent* of a *Body* circumſcribed by it, and preſſed equally above and below, without ſome other force to draw, or thruſt it downwards. But to demonſtrate the contrary of this Opinion, an *Experiment* was long ſince ſhewn before the *Royal Society*, whereby it appeared that the *Atmoſphere* was ſo far from being the Cauſe of *Gravity*, that the Effects thereof were much more Vigorous where the *preſſure* of the *Atmoſphere* was taken off; for a long *Glaſs-Receiver* having a light Down-feather included, being *evacuated* of *Air*, the Feather which in the *Air* would hardly ſink, did *in vacuo* deſcend with nearly the ſame *Velocity* as if it had been a Stone.

Some think to Illuſtrate this *Deſcent* of *Heavy Bodies*, by comparing it with the Vertue of the *Loadſtone*; but ſetting aſide the difference there is in the manner of their *Attractions*, the *Loadſtone* drawing only *in* and *about* its *Poles*, and the *Earth* near equally in all parts of its *Surface*, this Compariſon avails no more than to explain *ignotum per æque ignotum.*

Others aſſign a certain *Sympathetical attraction* between the *Earth* and its *Parts*, whereby they have, as it were, a deſire to be united, to be the Cauſe we enquire after: But this is ſo far from explaining the *Modus*, that it is little more, than to tell us in other terms, that *heavy bodies deſcend*, becauſe they *deſcend.*

This I ſay, not that I can pretend to ſubſtitute any *Solution* of this Important *Philoſophical Problem*, that ſhall more happily explicate the *Appearances* of *Gravity*, only it may be ſerviceable to thoſe with whom the Credit of great *Authors* ſways much, and who too-readily aſſent *in verba magiſtri*, to let them ſee that their Books are not always infallible: Beſides the detection of Errors is the firſt and ſureſt ſtep towards the diſcovery of Truth.

Tho' the Efficient Cauſe of *Gravity* be ſo obſcure, yet the final Cauſe thereof is clear enough; for it is by this ſingle *Principle* that the *Earth* and all the *Celeſtial Bodies* are kept from *diſſolution*: the leaſt of their *Particles* not being ſuffered to recede far from their *Surfaces*, without being immediately brought down again by vertue of this *Natural tendency*, which for their Preſervation, the Infinite Wiſdom of their *Creator* has Ordained to be towards each of their *Centers*;

nor can the *Globes* of the *Sun* and *Planets* otherwiſe be deſtroyed, but by taking from them this power of keeping their parts united.

The Affections or Properties of *Gravity*, and its manner of acting upon *Bodies falling*, have been in a great meaſure diſcovered, and moſt of them made out by *Mathematical demonſtration* in this our *Century*, by the accurate diligence of *Galilæus*, *Torricellius*, *Hugenius*, and others, and now lately by our worthy Country-man Mr. *Iſaac Newton*, (who has an incomparable *Treatiſe* of *Motion* almoſt ready for the *Preſs*) which Properties it may be very material here to enumerate, that they may ſerve for a Foundation to all thoſe that ſhall be willing to ſpend their Thoughts in ſearch of the true Cauſe of this *deſcent* of *Bodies*.

The firſt Property is, That by this principle of *Gravitation*, all Bodies do deſcend towards a Point, which either is, or elſe is very near to the Center of *Magnitude* of the *Earth* and *Sea*, about which the *Sea* forms it ſelf exactly into a *Spherical ſurface*, and the *Prominences* of the *Land*, conſidering the Bulk of the whole, differ but inſenſibly therefrom.

Secondly, That this Point or Center of *Gravitation*, is fixt within the *Earth*, or at leaſt has been ſo, ever ſince we have any *Authentick Hiſtory*: For a Conſequence of its Change, tho' never ſo little, would be the over-flowing of the low Lands on that ſide of the *Globe* towards which it approached, and the leaving new Iſlands bare on the oppoſite ſide, from which it receded; but for this Two thouſand years it appears, that the low Iſlands of the *Mediterranean* Sea (near to which the ancienteſt Writers lived) have continued much at the ſame height above the Water, as they now are found; and no *Inundations* or *Receſſes* of the *Sea* arguing any ſuch Change, are Recorded in Hiſtory; excepting the *Univerſal Deluge*, which can no better way be accounted for, than by ſuppoſing this Center of *Gravitation* removed for a time, towards the middle of the then inhabited parts of the World;

and a change of its place, but the two thouſandth part of the *Radius* of this *Globe*, were ſufficient to bury the Tops of the higheſt Hills under water.

Thirdly, That in all parts of the *Surface* of the *Earth*, or rather in all Points equidiſtant from its *Center*, the force of *Gravity* is nearly equal; ſo that the length of the *Pendulum* vibrating *ſeconds of time*, is found in all parts of the World to be very near the ſame. 'Tis true at S. *Helena* in the *Latitude* of 16 Degrees *South*, I found that the *Pendulum* of my Clock which vibrated *ſeconds*, needed to be made ſhorter than it had been in *England* by a very ſenſible ſpace, (but which at that time I neglected to obſerve accurately) before it would keep time; and ſince the like Obſervations has been made by the *French Obſervers* near the *Equinoctial*: Yet I dare not affirm that in mine it proceeded from any other Cauſe, than the great height of my place of Obſervation above the *Surface* of the *Sea*, whereby the *Gravity* being diminiſhed, the length of the *Pendulum* vibrating *ſeconds*, is proportionably ſhortned.

Fourthly, That *Gravity* does equally affect all *Bodies*, without regard either to their *matter*, *bulk*, or *figure*; ſo that the Impediment of the *Medium* being removed, the moſt compact and moſt looſe, the greateſt and ſmaleſt *Bodies* would deſcend the ſame *ſpaces* in equal times; the truth whereof will appear from the *Experiment* I beford cited. In theſe two laſt particulars, is ſhewn the great difference between *Gravity* and *Magnetiſm*, the one affecting only *Iron*, and that towards its *Poles*, the other all *Bodies* alike in every part. As a *Corollary*; from hence it will follow, that there is no ſuch thing as *poſitive levity*, thoſe things that appear light, being only comparatively ſo; and whereas ſeveral things riſe and ſwim in *fluids*, 'tis becauſe bulk for bulk, they are not ſo heavy as thoſe *fluids*; nor is there any reaſon why *Cork*, for inſtance, ſhould be ſaid to be light becauſe it ſwims on Water, any more than *Iron* becauſe it ſwims on *Mercury*.

Fifthly, That this power encreaſes as you deſcend, and de-

decreaſes as you aſcend from the Center, and that in the proportion of the *Squares* of the *diſtances* therefrom *reciprocally*, ſo as at a double diſtance to have but a quarter of the force; this property is the principle on which Mr. *Newton* has made out all the *Phænomena* of the *Cæleſtial Motions*, ſo eaſily and naturally that its truth is paſt diſpute. Beſides that, it is highly rational, that the *attractive* or *gravitating* power ſhould exert it ſelf more vigirouſly in a ſmall Sphere, and weaker in a greater, in proportion as it is contracted or expanded, and if ſo, ſeeing that the *ſurfaces* of *Spheres* are as the *Squares* of their *Radii*, this power at ſeveral diſtances will be as the *Squares* of thoſe *diſtances Reciprocally*, and then its whole action upon each *Spherical Surface*, be it great or ſmall will be alwaies equal. And this is evidently the rule of *Gravitation* towards the *Centers* of the *Sun*, *Jupiter*, *Saturn* and the *Earth*, and thence is reaſonably inferred, to be the general principle obſerved by *Nature*, in all the reſt of the *Celeſtial Bodies*.

Theſe are the principal affections of *Gravity*, from which the rules of the *fall* of *Bodies*, and the *motion* of *Projects* are *Mathamatically* deducible. Mr. *Iſaac Newton* has ſhewed how to define the ſpaces of the *deſcent* of a *Body*, let fall from any given highth, down to the *Center*. Suppoſing the *Gravitation* to increaſe, as in the fifth Property; but conſidering the ſmalneſs of hight, to which any *Project* can be made aſcend, and over how little an *Arch* of the *Globe* it can be caſt by any of our *Engines*, we may well enough ſuppoſe the *Gravity* equal throughout, and the deſcents of *Projects* in parralel lines, which in truth are towards the *Center*, the difference being ſo ſmall as by no means to be diſcovered in *Practice*. The *Oppoſition* of the *Air*, 'tis true, is conſiderable againſt all light bodies moving through it, as likewiſe againſt ſmall ones (of which more hereafter) but in great and ponderous Shot, this Impediment is found by *Experience* but very ſmall, and may ſafely be neglected.

Pro-

Propoſitions concerning the Deſcent of heavy Bodies, and the Motion of Projects.

Prop. I. The *Velocities* of *falling Bodies*, are proportionate to the times from the beginning of their *falls*.

This follows, for that the action of *Gravity* being *continual*, in every ſpace of time, the falling Body receives a new impulſe, equal to what it had before, in the ſame ſpace of time, received from the ſame power: For inſtance, in the firſt ſecond of time, the falling Body has acquired a *Velocity*, which in that time would carry it to a certain diſtance, ſuppoſe 32 foot, and were there no new force, would deſcend at that rate with an *equable Motion*; but in the next ſecond of time, the ſame power of *Gravity* continually acting thereon, ſuperadds a new *Veloſity* equal to the former; ſo that at the end of two ſeconds, the *Velecity* is double to what it was at the end of the firſt, and after the ſame manner may it be proved to be triple, at the end of the third ſecond, and ſo on. Wherefore the *Velcities* of *falling Bodies*, are proportionate to the times of their *falls*, *Q. E. D.*

Prop. II. The *Spaces* deſcribed by the *fall* of a Body, are as the *Squares* of the times, from the beginning of the *Fall*.

Demonſtration. Let A B (*Fig.*1. *Tab.*1.) repreſent the *time* of the *fall* of a *Body*, B C perpendicular to A B the *Veloſity* acquired at the end of the *fall*, and draw the line A C, then divide the line A B repreſenting the time into as many equal parts as you pleaſe, as b, b, b, b, &c. and through theſe points draw the lines bc, bc, bc, bc, &c. parallel to B C, 'tis manifeſt that the ſeveral lines, bc, repreſent the ſeveral *Velcities* of the falling Body, in ſuch parts of the *time* as A b is of A B, by the former propoſition. It is evident likewiſe that the *Area* A B C is the ſum of all the lines bc being taken, according to the method of *Indiviſibles*, infinitely many; ſo that the *Area* ABC repreſents the ſum of all the *Velocities*, between none and B C ſuppoſed infinitely many; which ſum is the ſpace deſended in the time repreſented by A B. And by the ſame reaſon the *Areas* A b c, will repreſent the ſpaces deſcended in the times A b; ſo then the ſpaces deſcended in the times AB, Ab, are as the *Areas* of the *Triangles* A B C, A b c, which by the 20th of the 6 of *Euclid* are as the *Squares* of their *Homologous ſides* A B, A b, that is to ſay, of the *Times*: wherefore the deſcents of *falling Bodies*, are as the *Squares* of the times of their *fall*, *Q. E. D.*

Prop. III. The *Velocity* which a *falling Body* acquires in any ſpace of time, is double to that, wherewith it would have moved the ſpace, deſcended by an equable motion, in the ſame *time*.

Demonſtration, Draw the line E C parallel to A B and A E parallel to B C in the ſame *fig.* 1. and compleat the *Parallelogram* A B C E, it is evident that the *Area* thereof may repreſent the ſpace, a *Body* moved equably with the *Velocity* B C, would deſcribe in the time A B, and the *Triangle* A B C repreſents the *ſpace* deſcribed by the *fall* of a *Body*, in the ſame time A B, by the ſecond propoſition. Now the *Triangle* A B C is half of the *Parallelogram* A B C E, and conſequently the ſpace deſcribed by the *fall*, is half what would have been deſcribed by an *equable Motion* with the *Velocity* B C, in the ſame time; wherefore the *Velocity* B C at the end of the *fall*, is double to that *Velocity*, which in the time A B, would have deſcribed the *ſpace fallen*, repreſented by the *Triangle* A B C, with an *equable Motion*, *Q. E. D.*

Prop. IV. All *Bodies* on or near the ſurface of the *Earth*, in their *fall*, deſcend ſo, as at the end of the firſt ſecond of time, they have deſcribed 16 feet one inch *London Meaſure*, and acquired the *Velocity* of 32 feet two inches in a ſecond.

This is made out from the 25th propoſition of the ſecond part of that Excellent Treatice of Mr. *Hugenius de Horologio Oſcillatorio*; wherein he demonſtrates the time of the leaſt *Vibrations* of a *Pendulum*, to be to the time of the *fall* of a *Body*, from the height of half the length of the *Pendulum*, as the *Circumference* of a *Circle* to its *Diameter* ; whence as a *Corollary* it follows, that as the *Square* of the *Diameter* to the *Square* of the *Circumference*, ſo half the length of the *Pendulum* vibrating *ſeconds*, to the *ſpace* deſcribed by the *fall* of a *body* in a *ſecond* of *time* : and the length of the *Pendulum* vibrating *ſeconds*, being found 39,125,or $\frac{1}{8}$ Inches, the *deſcent* in a *ſecond* will be found by the aforeſaid *Analogy* 16 Foot and one Inch, and by the third Propoſition , the *Velocity* will be double thereto ; and near to this it hath been found by ſeveral *Experiments*, which by reaſon of the *ſwiftneſs* of the *fall*, cannot ſo exactly determine its *quantity*. The Demonſtration of *Hugenius* being the Concluſion of a long train of *Conſequences*. I ſhall for brevity ſake omit ; and refer you to his Book, where theſe things are more amply treated of.

From theſe four *Propoſitions*, all *Queſtions* concerning the *Perpendicular fall of bodies*, are eaſily *ſolved*, and either *Time*, *Height*, or *Velocity* being aſſigned, one may readily find the other two. From them likewiſe is the Doctrine of *Projects* deducible, aſſuming the two following *Axioms* ; *viz.* That a *body* ſet a moving, will move on continually in a right *line* with an *equable motion*, unleſs ſome other force or impediment intervene, whereby it is accelerated , or retarded , or deflected.

Secondly, That a *Body* being agitated by two *motions* at a time, does by their *compounded forces* paſs through the ſame *points*, as it would do, were the two *motions divided* and acted *ſucceſſively*. As for inſtance, Suppoſe a *body* moved in the Line GF, (*Fig.*2.*Tab.*1.) from G to R, and there ſtopping, by another *impulſe* ſuppoſe it moved in a *ſpace* of *time* equal to the former, from R towards K, to V. I ſay, the *body* ſhall paſs through the point V, tho' theſe two *ſeveral forces*, acted both in the *ſame time*.

Prop. V. The *Motion* of all *Projects* is in the *Curve* of a *Parabola* : Let the *line* GRF (in *fig.* 2.) be the *line* in which the *Project* is directed, and in which by the firſt *Axiom* it would move equal *ſpaces* in equal *times*, were it not deflected downwards by the force of *Gravity*. Let G B be the *Horizontal line*, and G C a *Perpendicular* thereto. Then the *line* G R F being divided into equal parts, anſwering to equal *ſpaces* of *time*, let the *deſcents* of the *Project* be laid down in *lines parallel* to GC, proportioned as the *ſquares* of the *lines* GS, GR, GL, GF, or as the *ſquares* of the *times*, from S to T, from R to V, from L to X, and from F to B, and draw the *lines* TH, VD, XY, BC parallel to GF; I ſay the Points T,V, X,B, are Points in the *Curve* deſcribed by the *Project*, and that that *Curve* is a *Parabola*. By the ſecond *Axiom* they are Points in the *Curve*; and the parts of the *deſcent* GH, GD, GY, GC,= to ST, RV, LX, FB, being as the *ſquares* of the *times* (by the *ſecond Prop.)* that is, as the *ſquares* of the *Ordinates*, HT, DU, YX, BC, equal to GS, GR, GL, GF, the *ſpaces* meaſured in thoſe times; and there being no other *Curve* but the *Parabola*, whoſe parts of the *Diameter* are as the *ſquares* of the *Ordinates*, it follows that the *Curve* deſcribed by a *Project*, can be no other than a *Parabola*: And ſaying, as RU the *deſcent* in any *time*, to GR or UD the *direct motion* in the ſame *time*, ſo is U D, to a *third* proportional; that *third* will be the *line* called by all Writers of *Conicks*, the *Parameter* of the *Parabola* to the *Diameter* GC, which is alwaies the ſame in *Projects* caſt with the ſame *Velocity*: And the *Velocity* being defined by the number of *feet* moved in a *ſecond* of *time*, the *Parameter* will be found by dividing the *ſquare* of the *Velocity*, by 16 *feet* 1 *inch*, the *fall* of a *body* in the ſame *time*.

Lemma.

The *Sine* of the double of any *Arch*, is equal to twice the *Sine* of that *Arch* into its *Co-ſine*, divided by *Radius*; and the *Verſedſine* of the *double* of any *Arch* is equal to the *ſquare* of the *Sine* thereof divided by *Radius*.

Let the *Arch* BC (in *fig*.3.) be double the *Arch* BF, and A the *Center*; draw the *Radii* AB, AF, AC, and the *Chord*

BDC, and let fall BE perpendicular to AC, and the *Angle* EBC, will be equal to the *Angle* ABD, and the *Triangle* BCE, will be like to the *Triangle* BDA; wherefore it will be as AB to AD, ſo BC or twice BD; to BE, that is as *Radius* to *Co-ſine*, ſo twice *Sine*, to *Sine* of the double *Arch*. And as AB to BD, ſo twice BD or BC, to EC, that is as *Radius* to *Sine*, ſo twice that *Sine* to the *Verſed-ſine* of the double *Arch*; which two *Analogies* reſolved into *Equations*, are the *Propoſitions* contained in the *Lemma* to be proved.

Prop. VI. The *Horizontal* diſtances of *Projections* made with the ſame *Velocity*, at ſeveral *Elevations* of the *Line* of direction, are as the *Sines* of the doubled *Angles* of *Elevation*.

Let GB (*fig*.2.) the *Horizontal* diſtance be $= z$, the *ſine* of the *Angle* of *Elevation*, FGB, be $= s$, its *Co-ſine* $= c$, *Radius* $= r$, and the *Parameter* $= p$. It will be as c to s; ſo z to $\frac{sz}{c}$=FB=GC, and by reaſon of the *Parabola* $\frac{psz}{c}$=to the *ſquare* of CB, or GF,. Now as c to r, ſo is z to $\frac{zr}{c} =$ GF, and its *ſquare* $\frac{zzrr}{cc}$ will be therefore=to $\frac{psz}{c}$: which *Equation* reduced, will be $\frac{psc}{rr} = z$. But by the former *Lemma* $\frac{2sc}{r}$ is equal to the *Sine* of the double *Angle*, whereof s is the *Sine*: wherefore 'twil be as Radius to *Sine* of double the *Angle* FGB, ſo is half the *Parameter*, to the *Horizontal rang* or *diſtance* ſought; and at the ſeveral *Elevations*, the *ranges* are as the *ſines* of the double *Angles* of *Elevation* *Q.E.D.*

Corollary.

Hence it follows, that half the *Parameter* is the greateſt R*andon*, and that that happens at the *Elevation* of 45 degrees, the *ſine* of whoſe double is R*adius*,. Likewiſe that the R*anges* equally diſtant above and below 45 are equal, as are the *ſines* of all doubled *Arches*, to the *ſines* of their doubled *Complements*.

Prop. VII. The *Altitudes* of *Projections* made with the ſame *Velocity*, at ſeveral *Elevations*, are as the *verſed ſines* of the doubled *Angles* of *Elevation*: As c is to s :: ſo is $\frac{psc}{rr}$ $=$GB to $\frac{pss}{rr}=$BF, and UK$=$RU$=$ $\frac{\text{BF}}{4}$, the *Altitude* of the *Projection* $=\frac{pss}{4rr}$. Now by the foregoing *Lemma* $\frac{2ss}{r}=$to the *verſed ſine* of the double *Angle*, and therefore it will be as *Radius*, to *verſed ſine* of double the *Angle* FGB, ſo an *8th* of *Parameters* to the height of the *Projection* VK; and ſo theſe heights at ſeveral *Elevations* are as the ſaid *verſed ſines*, *Q. E. D.*

Corollary.

From hence it is plain,that the greateſt *Altitude* of the perpendicular *Projection* is a *4th* of *Parameter*,or half the greateſt *Horizontal* Rang; the *verſed ſine* of 180 degrees being $= 2r$.

Prop. VIII. The *Lines* GF, or times of the flight of a *Project* caſt with the ſame degree of *velocity* at different *Elevations*, are as the *ſines* of the *Elevations*.

As c is to r :: ſo is $\frac{psc}{rr}=$ GB by the *6 Prop.* to $\frac{ps}{r}=$GF, that is as *Radius* to *ſine* of *Elevation*, ſo the *Parameter* to the *line* GF; ſo the *lines* GF are as the *ſines* of *Elevation*, and the *Times* are proportional to thoſe *Lines*; wherefore the *Times* are as the *Sines* of *Elevation*: *Ergo conſtat propoſitio.*

Prop. IX. Problem. A *Projection* being made as you pleaſe, having the Diſtance and Altitude, or Deſcent of an *Object*, through which the *Project* paſſes, together with the *Angle* of *Elevation* of the *line* of *Direction*; to find the *Parameter* and *Velocity*, that is (in *Fig.* 2.) having the *Angle* FGB, GM, and MX.

Solution. As *Radius* to *Secant* of FGB, ſo GM the *diſtance* given,to GL; and as *Radius* to *Tangent* of FGB, ſo GM to LM. Then LM—MX in *heights*, or +MX in *deſcents*; or elſe MX—ML, if the *direction* be below the *Horizontal-line*, is the *fall* in the *time* that the direct *impulſe* given in G would have carried the *Project* from G to L=LX=GY; then by reaſon of the *Parabola*; as LX or GY, is to GL or YX, :: ſo is GL to the *Parameter* ſought. To find the *Velocity* of the *Impulſe*, by *Prop*.2. & 4, find the *time* in *ſeconds* that a *body* would fall the *ſpace* LX, and by that dividing the *line* GL, the *Quote* will be the *Velocity*, or *ſpace* moved in a *ſecond* ſought, which is alwaies a *mean proportional* between the *Parameter* and 16 *feet* 1 *inch*.

Prop. X. *Problem* 2. Having the *Parameter*, Horizontal diſtance, and height or deſcent of an *Object*, to find the *Elevations* of the *line* of *direction* neceſſary to hit the given *Object*; that is, having GM, MX, and the greateſt *Randon* equal to half the *Parameter*; to find the *Angles* FGB.

Let the *Tangent* of the *Angle* ſought be $=t$, the *Horizontal diſtance* GM$=b$, the Altitude of the *Object* MX$=h$, the *Parameter*$=p$, and *Radius*$=r$, and it will be,

As r to t, ſo b to $\frac{tb}{r}=$ML and $\frac{t\,b}{r}\mp h$ in aſcents / in deſcents $=$LX, and

$\frac{p\,t\,b}{r}\mp p\,h=$GL *quad.* $=$XY *quad. ratione Parabolæ*; but

$b\,b\mp\frac{tt\,bb}{r\,r}=$GL *quad.* 47. 1. *Euclid.* Wherefore

$\frac{p\,t\,b}{r}\mp p\,h=bb\mp\frac{t\,t\,b\,b}{r\,r}$ which Equation tranſpoſed, is

$\frac{t\,t\,b\,b}{r\,r}=\frac{p\,t\,b}{r}\mp p\,h-b\,b$, divided by $b\,b$ is

$\frac{tt}{rr}=\frac{pt}{br}\mp\frac{ph}{b\,b}-1$. this Equation ſhews the Queſtion to have two Anſwers, and the Roots thereof are $\frac{t}{r}=\frac{p}{2\,b}\mp\sqrt{\frac{p\,p\mp4\,p\,h}{4\,bb}-1}$ from which I derive the following Rule.

Divide half the *Parameter* by the Horizontal diſtance, and keep the Quote; *viz.* $\frac{p}{2b}$ then ſay, as *ſquare* of the *diſtance* given to the half *Parameter*, ſo half *Parameter* $\mp$ double height deſcent to the *ſquare* of a *Secant* $= \frac{pp \mp 4ph}{4bb}$ the *Tangent* anſwering to that *Secant*, will be $\sqrt{\frac{pp \mp 4ph}{4bb} - 1}$ or rr: ſo then the ſum and difference of the afore-found *Quote*, and this *Tangent* will be the Roots of the *Equation*, and the *Tangents* of the *Elevations* ſought.

Note here, that in *Deſcents*, if the *Tangent* exceed the *Quote*, as it does when ph is more than bb, the *direction* of the lower *Elevation* will be below the *Horizon*, and if $ph = bb$, it muſt be directed *Horizontal*, and the *Tangent* of the upper *Elevation* will be $\frac{pr}{b}$: Note likewiſe, that if $4bb + 4ph$ in *aſcents*, or $4bb - 4ph$ in *deſcents*, be equal to pp, there is but one *Elevation* that can hit the *Object*, and its *Tangent* is $\frac{pr}{2b}$ and if $4bb + 4ph$ in *aſcents*, or $4bb - 4ph$ in *deſcents*, do exceed pp, the *Object* is without the reach of a *Project* caſt with that *Velocity*, and ſo the thing impoſſible.

From this *Equation* $4bb \mp 4ph = pp$ are determined the utmoſt limits of the reach of any *Project*, and the Figure aſſigned, wherein are all the *heights* upon each *Horizontal diſtance* beyond which it cannot paſs; for by reduction of that *Equation*, h will be found $= \frac{1}{4}p - \frac{bb}{p}$ in *heights*, and $\frac{bb}{p} - \frac{1}{4}p$ in *deſcents*; from whence it follows, that all the Points h are in the *Curve* of the *Parabola*, whoſe *Focus* is the Point from whence the *Project* is caſt, and whoſe *Latus rectum*, or *Parameter ad Axem* is $= p$. Likewiſe from the ſame *Equation* may the leaſt *Parameter* or *Velocity* be found capable to reach the

Object propoſed; for $bb = \frac{1}{4}pp \mp ph$ being reduced $\frac{1}{2}p$ will be $=\sqrt{bb+hh} \pm h$ $\begin{matrix}\text{in aſcents}\\ \text{in deſcents,}\end{matrix}$ which is the *Horizontal rang* at 45 degrees, that would juſt reach the *Object*, and the *Elevation* requiſite will be eaſily had; for dividing the ſo found *Semi-parameter* by the *Horizontal diſtance* given b, the *Quote* into *Radius* will be the *Tangent* of the *Elevation* ſought. This Rule may be of good uſe to all *Bombardiers* and *Gunners*, not only that they may uſe no more Powder than is neceſſary, to caſt their *Bombs* into the place aſſigned, but that they may ſhoot with much more certainty, for that a ſmall Error committed in the *Elevation* of the *Piece*, will produce no ſenſible difference in the fall of the Shot: For which Reaſons the *French* Engineers in their late Sieges have uſed Morter-pieces inclined conſtantly to the *Elevation* of 45, proportioning their Charge of Powder according to the diſtance of the *Object* they intend to ſtrike on the *Horizon*.

And this is all that need to be ſaid concerning this *Problem*, of Shooting upon *hights* and *deſcents*. But if a *Geometrical* conſtruction thereof be required; I think I have one, that is as eaſy as any can be expected, which I deduce from the forgoing *Analytical Solution*, viz. $\frac{t}{r} = \frac{p}{2b} \pm \frac{\sqrt{\frac{1}{4}pp \pm ph - bb}}{bb}$, and tis this. Having made the right *Angle* L D A, *Tab.* 1. *fig.* 4. make D A, D F = p, or greateſt Rang, D G = b the *Horizontal* diſtance, and D B D C = h, the *Perpendicular hight* of the *Object*; and draw G B, and make DE = thereto. Then with the *Radius* A C and *center* E ſweep an Arch, which if the thing be poſſible, will Interſect the line A D in H; and the line D H being laid both waies from F will give the points K and L, to which draw the lines G L, G K; I ſay the *Angles* LGD, KGD are the Elevations required for hitting the *Object* B. But note that if B be below the *Horizon*, its *deſcent* D C = D B muſt be laid from A, ſo as to have A C = to A D + D C. *Note* likewiſe, that if in *deſcents* D H be greater than F D, and ſo K fall below D the

Angle KGD ſhall be the depreſſion below the *Horizon* : Now this Conſtruction ſo naturally follows from the *Equation*, that I ſhall need ſay no more about it.

Prop. XI. To determine the force or *Velocity* of a *Project*, in every point of the *Curve* it deſcribes.

To do this we need no other *præcognita*, but only the third Propoſition, Viz. that the *Velocity* of *falling Bodies*, is double to that which in the ſame time, would have deſcribed the ſpace *fallen* by an equable motion : For the *Velocity* of a Project, is compounded of the conſtant equal *Velocity* of the impreſſed motion, and the *Velocity* of the *fall*, under a given *Angle*, viz. the complement of the *Elevation* : For inſtance, in *Fig.* 2, in the time wherein a project would move from G to L, it deſcends from L to X, and by the third *Propoſition* has acquired a *Velocity*, which in that time would have carried it by an equable motion from L to Z or twice the deſcent L X; and drawing the line G Z, I ſay the *Velocity* in the point X, compounded of the *Velocities* G L and L Z under the *Angle* G L Z, is to the *Velocity* impreſt in the point G, as G Z is to G L ; this follows from our ſecond *Axiome* ; and by the 20 and 21.*Prop. lib.*1, *conic. Midorgii*, XO parallel and equal to G Z ſhall touch the *Parabola* in the point X. So that the *Velocities* in the ſeveral points, are as the lengths of the *Tangents* to the *Parabola* in thoſe points, intercepted between any two *Diameters* : And theſe again are as the *Secants* of the *Angles*, which thoſe *Tangents* continued make with the *Horizontal* line G B. From what is here laid down, may the comparative force of a *ſhott* in any two points of the *Curve*, be either *Geometrically* or *Arithmetically* diſcovered.

Corollary.

From hence it follows, that the force of a Shot is alwaies leaſt at U, or the *Vertex* of the *Parabola*, and that at equal diſtances therefrom, as at T and X, G and B its force is alwaies equal, and that the leaſt force in U is to that in G and

B, as *Radius* to the Secant of the *Angle* of *Elevation* F G B.

Theſe *Propoſitions* conſidered, there is no queſtion relating to *Projects*, which by the help of them may not eaſily be Solved; and tho' it be true that moſt of them are to be met withal, in *Galileus, Torricellius* and others, who have taken them from thoſe Authors, yet their Books being Forreign, and not eaſy to come by, and their *Demonſtrations* long and difficult, I thought it not amiſs to give the whole *Doctrine* here in *Engliſh*, with ſuch ſhort *Analytical* Proof of my own, as might be ſufficient to evince their Truth.

The Tenth *Propoſition* containes a *Problem*, untouch't by *Torricellius*, which is of the greateſt uſe in *Gunnery*, and for the ſake of which this *Diſcourſe* was principally intended; It was firſt Solved by Mr. *Anderſon*, in his Book of the Genuine uſe and effects of the *Gunn*, Printed in the Year 1674; but his Solution required ſo much Calculation, that it put me upon ſearch, whether it might not be done more eaſily, and thereupon in the Year 1678 I found out the rule I now publiſh, and from it the *Geometrical* Conſtruction: Since which time there has a large *Treatiſe* of this Subject Entituled, *L'art de jetter les Bombes*, been Publiſhed in *France* by *Monſieur Blondel*, wherein he gives the Solutions of this *Problem* by *Meſſieurs Bout*, *Romer* and *de la Hire*; But none of them being the ſame with mine, or in my Opinion more eaſy, and moſt of them more Operoſe, and beſides mine finding the *Tangent*, which generally determines the *Angle* better than its *Sine*, I thought my ſelf obliged to Print it for the uſe of all ſuch, as deſire to be informed in the *Mathematical* part, of the Art of *Gunnery*.

Now theſe rules were rigidly true, were it not, as I ſaid before, for the Oppoſition of the Medium, whereby not only the direct impreſt Motion is continually retarded, by likewiſe the increaſe of the *Velocity* of the *fall*, ſo that the ſpaces deſcribed thereby, are not exactly as the ſquares of the times: But what this Oppoſition of the *Air* is, againſt ſeveral *Velocities*, *Bulks*, and *Weights*, is not ſo eaſy to determine. 'Tis certain that the weight of *Air*, to that of *Water*, is nearly as 1 to 800, whence the weight thereof, to that of any *Project* is given; tis very likely, that to the ſame *Velocity*, and *Magnitude*, but of different matter, the *Oppoſition* ſhould be *reciprocally* as the weights of the ſhott; as likewiſe that to ſhott of the ſame *Velocity* and matter, but of different Sizes, it ſhould be as the *Diameters reciprocally*: whence generally the *Oppoſition* to ſhott with the ſame *Velocity*, but of differing *Diameters*, and *Materials*, ſhould be as their *Specifick Gravities* into their *Diameters reciprocally*; but whether the *Oppoſition*, to differing *Velocities* of the ſame ſhott, be as the *Squares* of thoſe *Velocities*, or as the *Velocities* themſelves, or otherwiſe, is yet a harder Queſtion. However it be, tis certain, that in large Shott of *Mettal*, whoſe weight many Thouſand times Surpaſſes that of the *Air*, and whoſe force is very great, in proportion to the *Surface* wherewith they preſs thereon; this *Oppoſition* is ſcarce diſcernable: For by ſeveral *Experiments* made with all Care and Circumſpection with a *Morterpeice* Extraordinary well fixt to the *Earth* on purpoſe, which carried a Solid Braſs Shott of $4\frac{1}{5}$ Inches *Diameter*, and of about 14 Pound weight, the *Ranges* above and below 45 *Degrees* were found nearly equal; if there were any difference, the under *Ranges* went rather the fartheſt, but thoſe differences were uſually leſs than the Errours committed in ordinary *Practice*, by the unequal Goodneſs and Dryneſs of the ſame ſort of *Pouder*, by the Unfitneſs of the *Shott* to the *Bore*, and by the Looſneſs of the Carriage.

In a Smaller *Braſs-Shott* of about an Inch and half *Diameter*, caſt by a *Croſs-Bow* which *ranged* it, at moſt about 400 *foot*, the Force being much more Equal than in the *Morterpeice*, this difference was found more Curiouſly, and Conſtantly and moſt Evidently, the under *Ranges* out went the upper. From which Trials I conclude, that altho' in ſmall and light *Shott*, the *Oppoſition* of the *Air*, ought and muſt be accounted for; yet in *Shooting* of Great and Weighty *Bombs*, there need be very little or no allowance made; and ſo theſe *Rules* may be put in *Practice* to all Intents and Purpoſes, as if this *Impediment* were abſolutely Removed.

An Account of an Experiment *ſhewn before the* Royal Society, *of Shooting by the Rarefaction of the Air: By* Dr. D. Papin, R. S. S.

WHereas ordinary Wind-Guns do their *Effect* by the Compreſſion of the Air. *Ottho Ghericke* hath found a new Sort that ſhoots by Rarefaction; and he hath Publiſht that device at large in his Book about *Pneumatick Experiments*, but he doth not expreſs how ſtrong was the Effect. I have therefore had the Curioſity to try it my ſelf by another Contrivance, which I take to be better than his: Firſt, becauſe I can make a Rarefaction much more perfect than he could do. Secondly, becauſe his Device could not be uſed but for Guns of a ſmall bore; but my way may be apply'd to the biggeſt bore that can be made by Workmen: So that one might by this means throw up vaſt Weights to a great diſtance.

A A is a Pipe very equal from one end to the other.

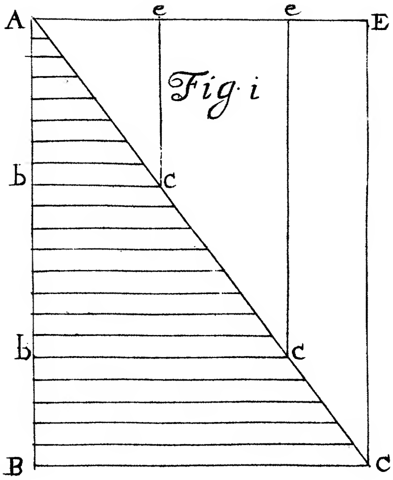
B B a ſmall Pipe ſoder'd to a Hole near the end of the Pipe A A, and apply'd to the Plate of the *Pneumatick Engine*.

C C C C ſome kind of Stool to bear up the hinder part of the Pipe A A.

D. a peice of Lead fitted to the bore of the pipe A A.

The pipe A A is to be ſhut at both ends by *Valves* outwardly apply'd, and ſo the ſaid pipe A A, though never ſo big, may be exhauſted of Air by means of the *Pneumatick Engine*: Which done, the *Valve* towards D muſt be ſuddenly open'd, ſo that the whole preſſure of the *Atmoſphere* acting upon the Lead D may drive it along the pipe A A with

Tab 1.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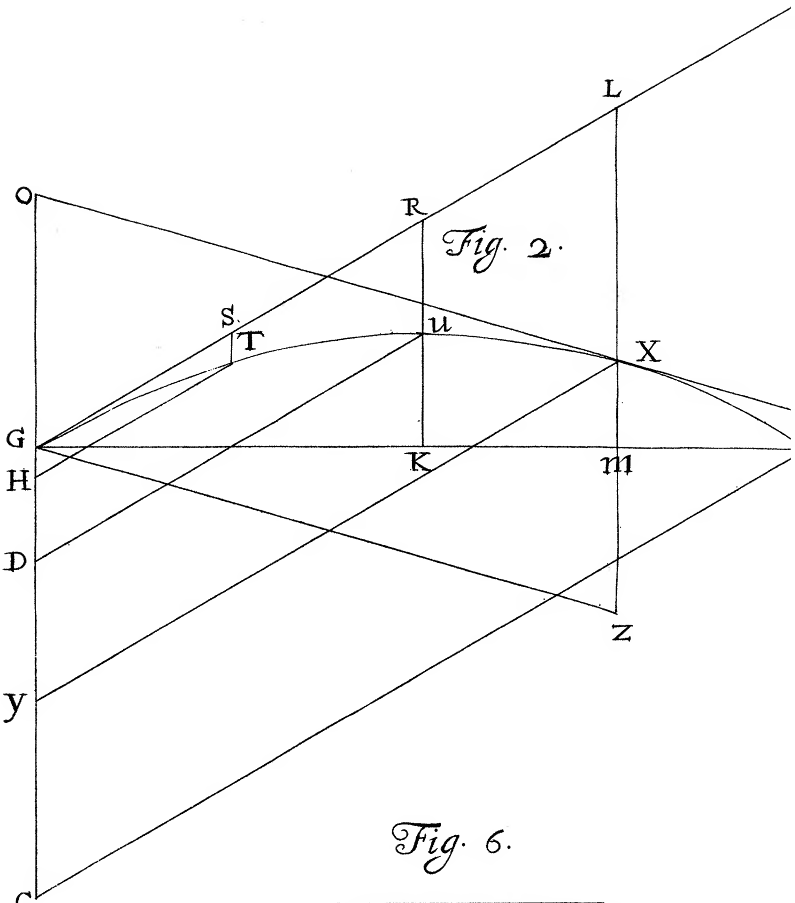
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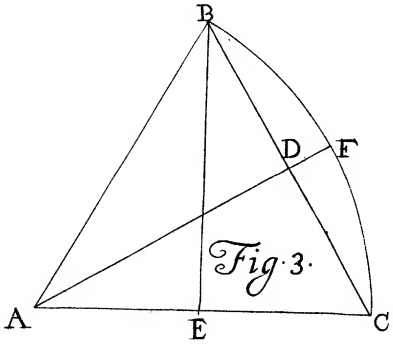
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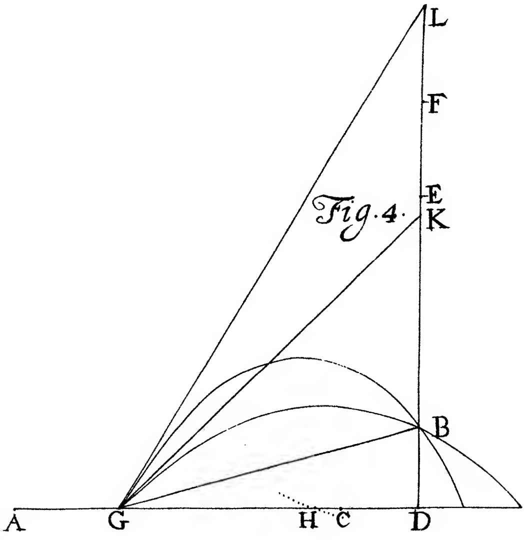
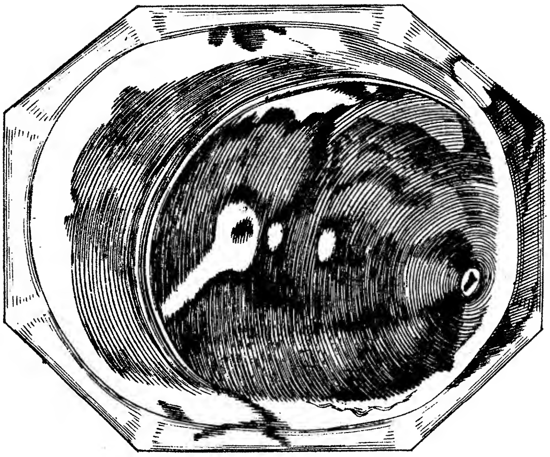


Fig. 4.

Fig. 6.



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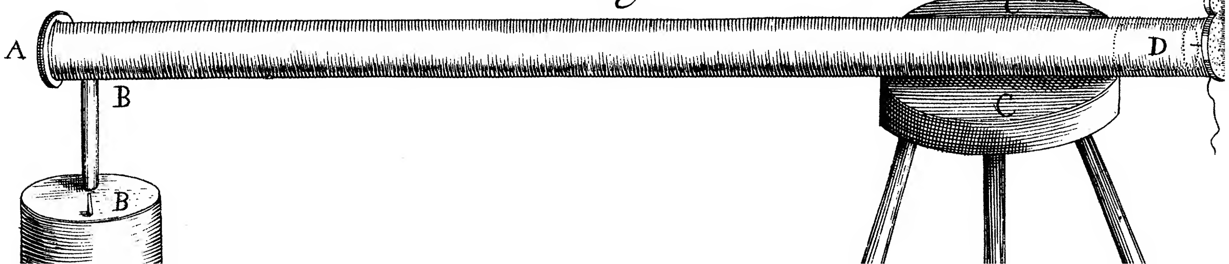

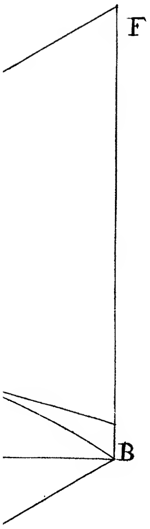
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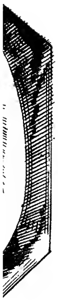

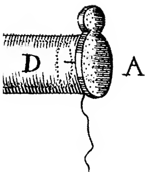
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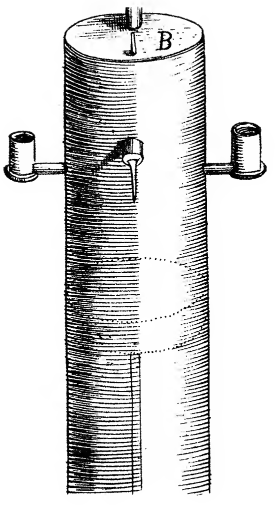
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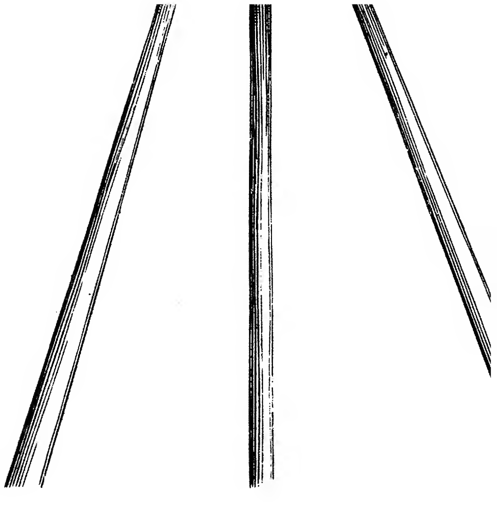

Tab 1

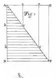

Fig 1
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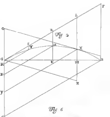

Fig 2
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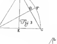

Fig 3
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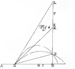

Fig 4
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Fig 6

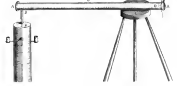

Fig 5
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